

ELLIPTIC KRV

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PRIMES Conference

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THE ASSOCIATIVE ALGEBRA

$$x \text{ --- } y \text{ --- } y \text{ --- } x \text{ --- } x \text{ --- } x \text{ --- } y$$

corresponds to

$$xyyxxy \quad (\neq xxxxyy)$$

THE ROOTED LIE TREE



corresponds to

$$[[y, x], y]$$

$$\neq [y, [x, y]]$$

THE LIE TREE, or $F(L)$

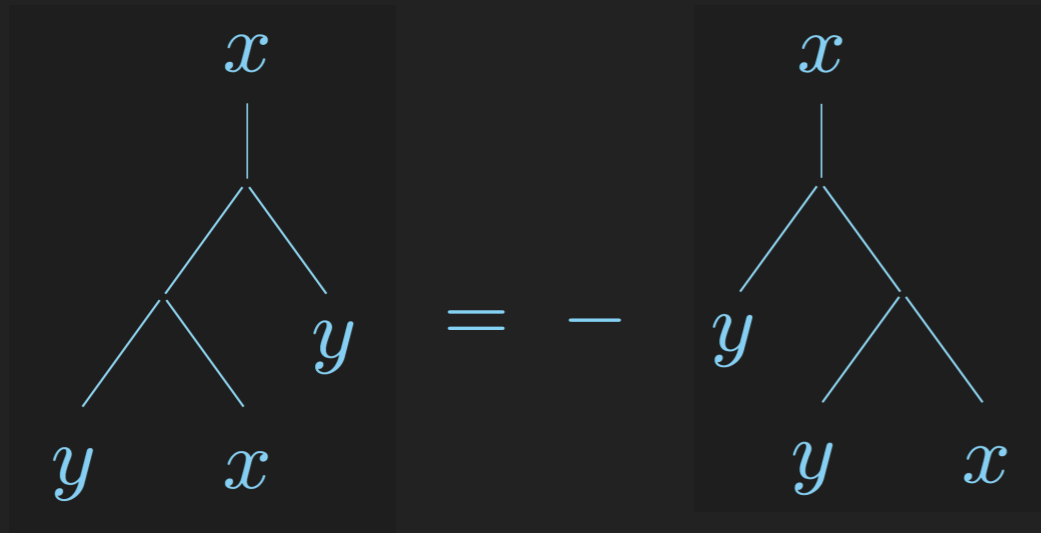


corresponds to
a *derivation*:

$$u(x) = 2[x, [y, x]]$$

$$u(y) = -2[[y, x], y]$$

PROPERTIES OF THE LIE TREE



The Lie Brackets $[\cdot, \cdot]$ gives a bilinear operation that satisfies:

1. **(Antisymmetry)**

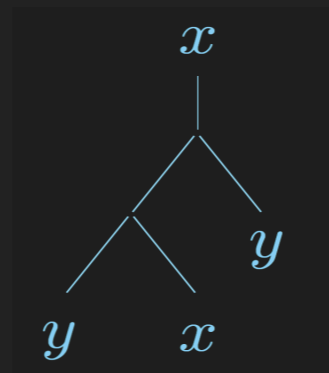
$$[x, y] = -[y, x]$$

2. **(Jacobi Identity)**

$$[x, [y, z]] + [z, [x, y]] + [y, [z, x]] = 0$$

GRAPHICAL REPRESENTATIONS

$$x - y - y - x - x$$



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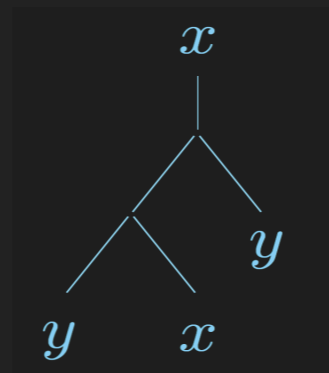
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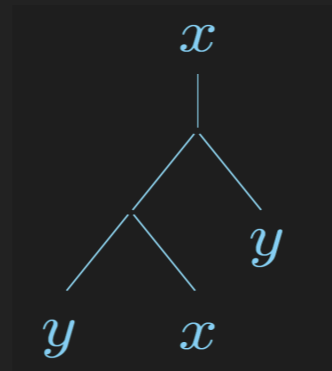
DEF: \mathfrak{krv}

$$\mathfrak{krv} = \mathfrak{krv}^{(1,1)} = \{u \in \mathfrak{tder}(1,1) = \text{Der}^+(L(x,y)) \mid u([x,y]) = 0, \text{div}(u) = 0\}$$

We'll understand the definition in terms of *graphs*.

GRAPHICAL
REPRESENTATIONS

$$x - y - y - x - x$$



$$\ker \text{div} = \{ \text{der } u \mid u([x, y]) = 0, \text{div}(u) = 0 \}$$

Graphical Interpretation of $u([x, y]) = 0$

Recall:



corresponds to

a *derivation*:

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GRAPHICAL
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$$x - y - y - x - x$$

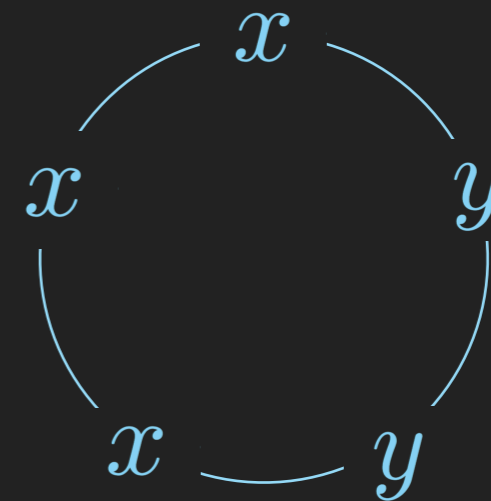


THE TRACE

Recall that $x - y - y - x - x$

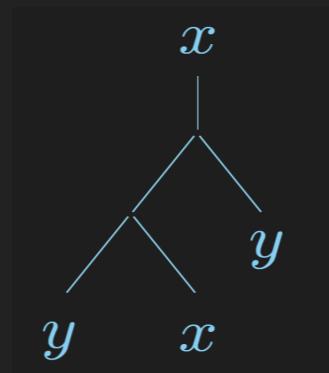
is a term in the Associative Algebra

$$\text{tr}(x - y - y - x - x) =$$



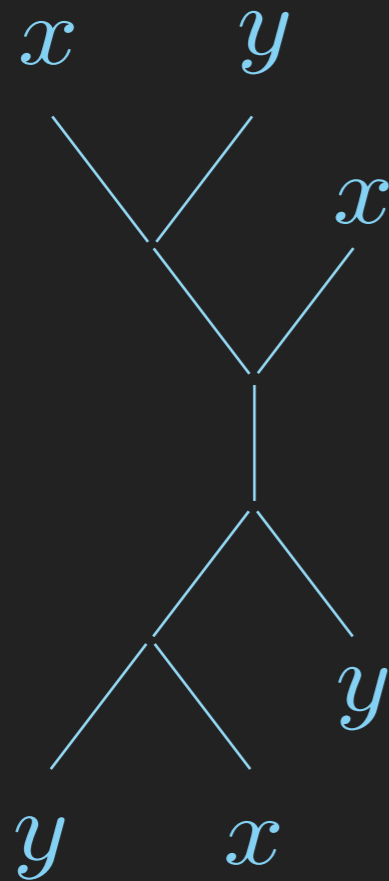
GRAPHICAL REPRESENTATIONS

$$x - y - y - x - x$$



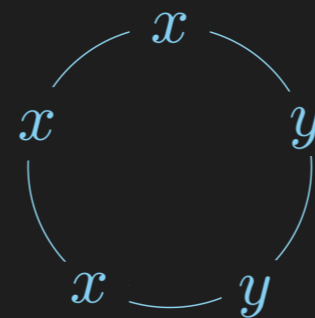
$$\ker \text{div} = \{ \text{der } u \mid u([x, y]) = 0, \text{div}(u) = 0 \}$$

Graphical Interpretation of div



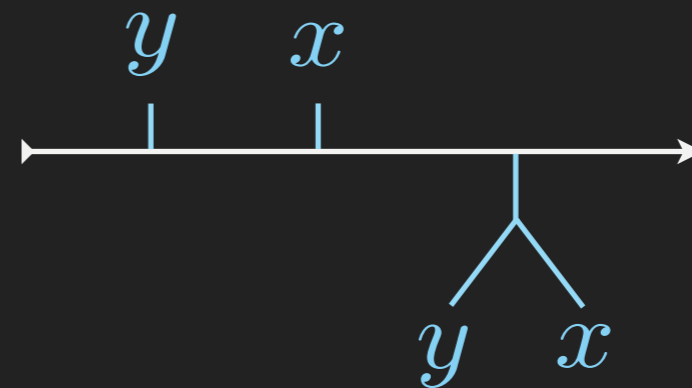
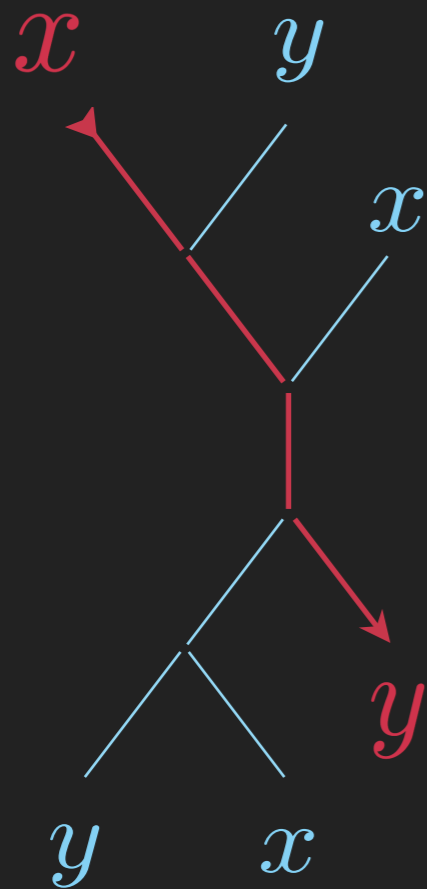
GRAPHICAL REPRESENTATIONS

$$x - y - y - x - x$$

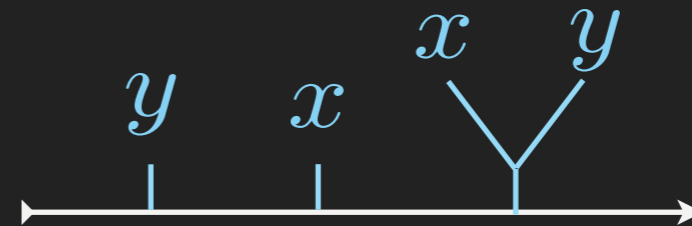


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Graphical Interpretation of div

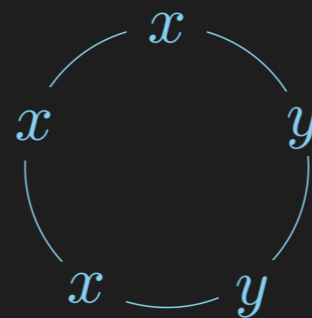


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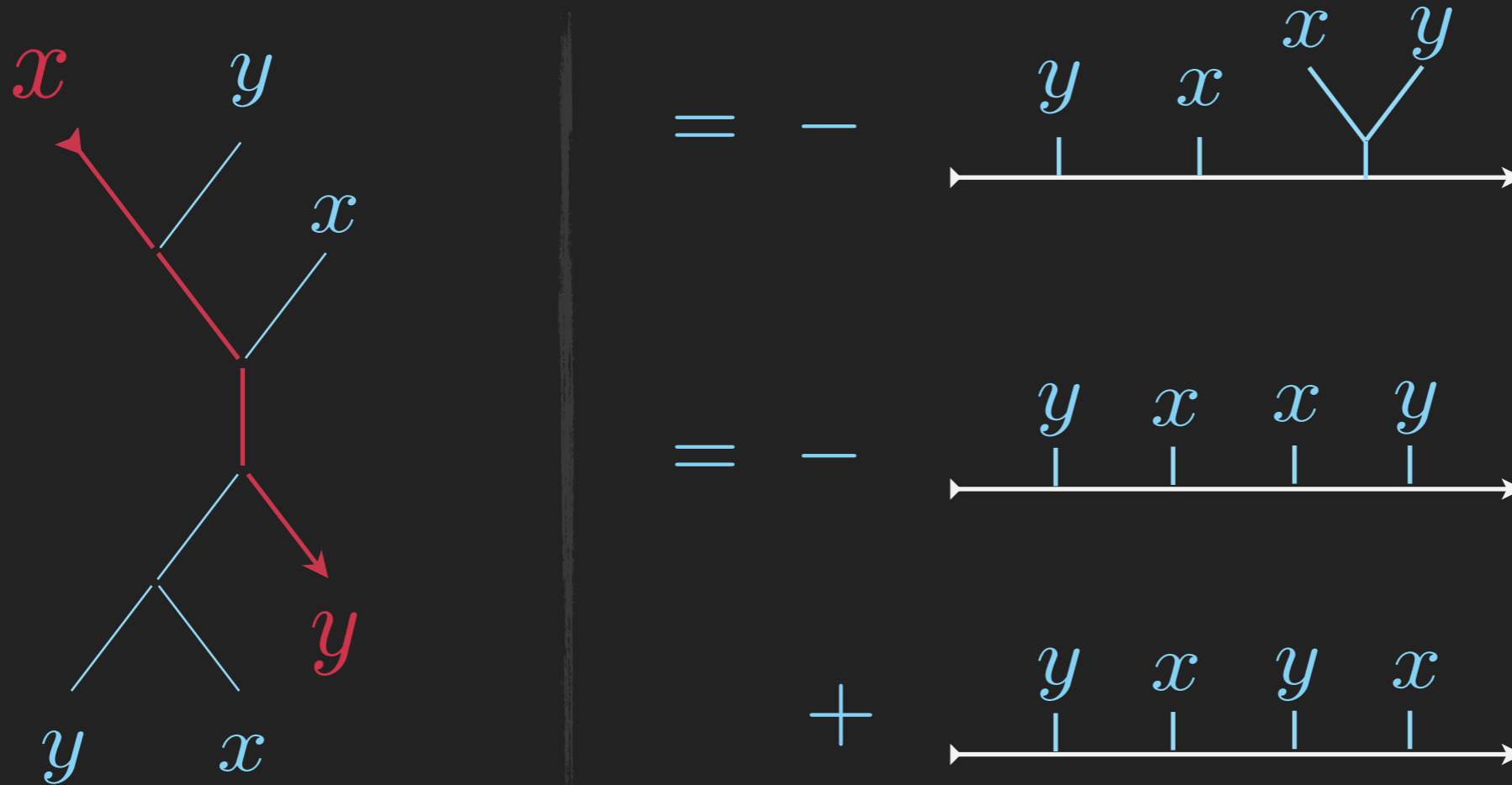
GRAPHICAL
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$x - y - y - x - x$



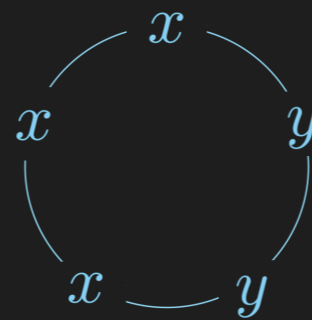
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Graphical Interpretation of div



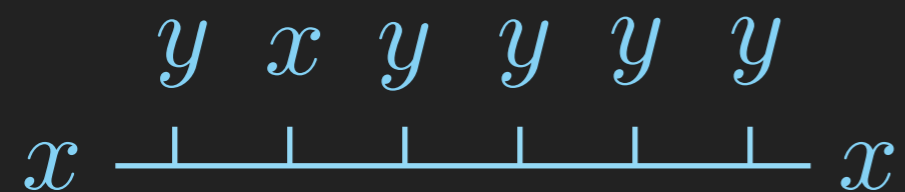
GRAPHICAL REPRESENTATIONS

$$x - y - y - x - x$$



CURRENT PROGRESS

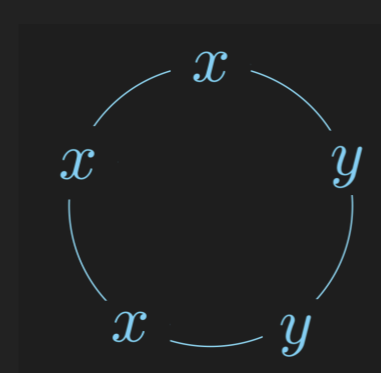
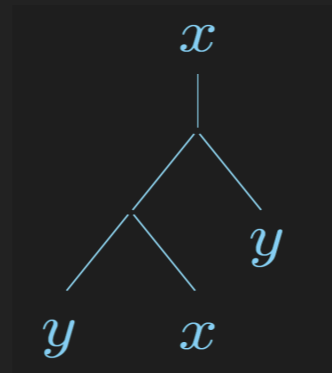
Def: the birds-on-a-wire graph



Claim:

Every tree can be rewritten as birds-on-a-wire graph

GRAPHICAL REPRESENTATIONS



PROGRESS AND GOALS

BASIS FOR $F(L)^{\leq 7}$

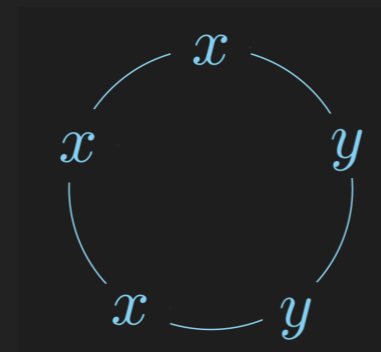
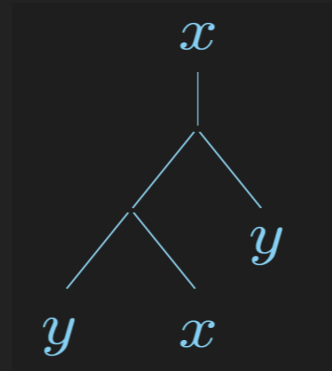
$$x - x \quad x - y \quad y - y \quad \Bigg| \quad x \begin{array}{c} y \quad y \\ \hline \end{array} x$$

$$x \begin{array}{c} y \quad y \quad y \quad y \\ \hline \end{array} x \quad x \begin{array}{c} y \quad x \quad x \quad y \\ \hline \end{array} x \quad x \begin{array}{c} y \quad x \quad y \quad y \\ \hline \end{array} x$$

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GRAPHICAL REPRESENTATIONS

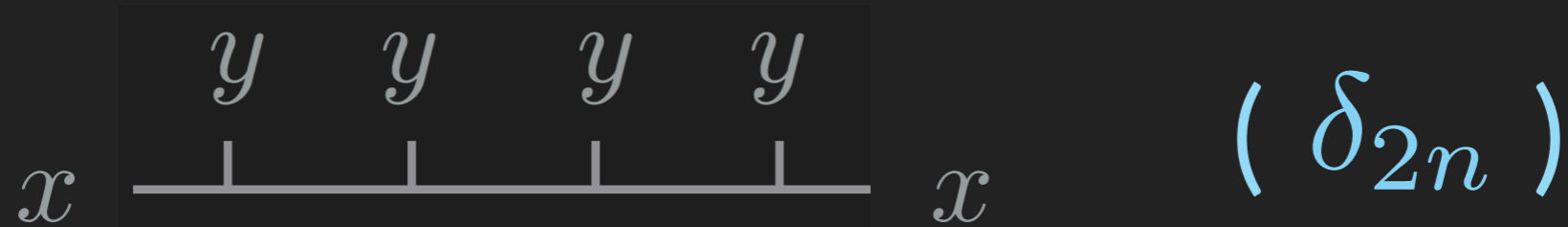
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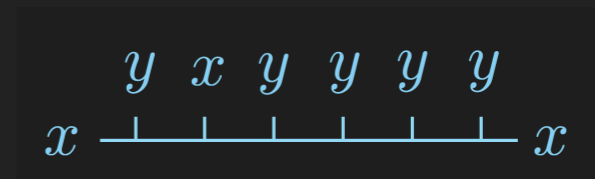
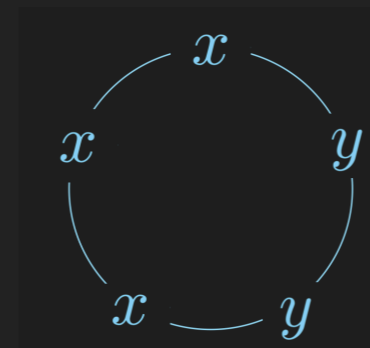
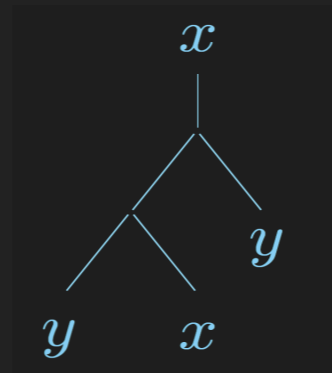
MORE SMALL ELEMENTS

- ▶ Elements in krv with 2 x 's



- ▶ Elements in krv with 3 x 's (?)

GRAPHICAL REPRESENTATIONS



GOALS

- ▶ Study the elements of \mathbb{F}_q with small total number of x and y 's.
 - ▶ Possibly using a computer
- ▶ Study the elements of \mathbb{F}_q with small number of x 's.
Currently, we are working on elements with 3 x 's.

ACKNOWLEDGEMENTS

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